Toward a Rotating Embedded Braneworld Cosmology:

Critical Assessment, Formal Viability, and a Research Programme for Geometric Dark Sectors

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Scope and epistemic status. This manuscript is written as a literature-grounded critical synthesis of an exploratory higher-dimensional cosmology. It does *not* claim that a complete unified theory has already been derived. Instead, it identifies (i) the closest existing work, (ii) the mathematically surviving channel after the strongest objections are applied, (iii) the sharpest no-go results, and (iv) the concrete calculations required for the framework to become a viable research programme.

Abstract

We examine a cosmological proposal in which the observable universe is interpreted as a three-dimensional boundary or brane associated with a rotating and expanding higher-dimensional geometry. In its strongest original form, the proposal sought to unify dark matter, dark energy, emergent time, and quantum nonlocality through a single geometric premise: a rotating, expanding hyperspherical universe embedded in a five-dimensional ambient space. The present manuscript subjects that programme to a mechanism-level literature review and a formal viability analysis. Three parts of the original proposal are found to be untenable in their literal form: dark matter as direct global rigid rotation is ruled out by cosmic isotropy and vorticity bounds; a purely gravitomagnetic or frame-dragging explanation of galactic rotation curves is far too weak; and the quantum-nonlocality claim lacks a mechanism capable of reproducing standard quantum limits. After these eliminations, one mathematically nontrivial channel remains: bulk angular momentum can enter the effective four-dimensional brane equations through the nonlocal Weyl sector, with local extrinsic-curvature gradients sourcing a magnetic-to-electric Weyl transfer. Because Friedmann-Robertson-Walker symmetry suppresses the vector and anisotropic Weyl pieces at background level, this channel can in principle remain safe with respect to cosmic microwave background isotropy while becoming active near local inhomogeneities. We show, however, that the surviving programme is not a one-term unification. The literature instead points to a sectoral structure: a homogeneous scalar sector can mimic pressureless matter in certain scalar-gravity or linear-dilaton bulk completions; a distinct vector or off-diagonal sector carries the leading slow-rotation response; and dark-energy-like behaviour belongs to a separate background scalar sector associated with brane motion, extrinsic curvature, or a different fluid regime. The strongest viable formulation is therefore a *single bulk theory with multiple effective sectors*, not a single geometric correction that simultaneously performs every cosmological task. We conclude by presenting a falsifiable research roadmap, centred on a rotating extension of linear-dilaton braneworld cosmology and on the calculation of the induced galactic halo and lensing response.

Keywords: braneworld cosmology; higher-dimensional gravity; Weyl fluid; extrinsic curvature; rotating bulk; Kerr–AdS; linear dilaton; geometric dark matter; dark energy; embedding gravity.

1. Introduction

The modern dark sector problem remains bifurcated. On the one hand, late-time cosmic acceleration is commonly represented by a cosmological constant or a dynamical dark-energy component. On the other hand, galactic, cluster, and cosmological observations require a matter component that is dynamically cold, effectively collisionless on large scales, and already present before recombination. In parallel with particle-based dark matter and vacuum-energy approaches, a persistent line of research has attempted to reinterpret part or all of the dark sector as an effect of geometry: modified gravity, induced gravity, embedding theory, braneworld cosmology, mirage cosmology, and holographic or extra-dimensional effective fluids all belong to this general family [1, 2, 3, 4].

The hypothesis evaluated here belongs squarely to that geometric class, but in a more ambitious form. Its original version proposed that the observable universe should be treated as the boundary of a rotating and expanding higher-dimensional object, heuristically described as a hypersphere embedded in a five-dimensional ambient space. Within that picture, galactic dark matter was to be reinterpreted as a projection of higher-dimensional rotation, cosmic acceleration as the manifestation of motion or drift into the fifth dimension, time as an emergent ordering of geometric change, and quantum nonlocality as a projection artefact arising from higher-dimensional adjacency.

This manuscript has a narrower and more technical goal. It asks four questions.

1. What is the closest existing literature to each mechanism, not merely by analogy but by explicit field content, geometry, and phenomenology?
2. Which pieces of the proposal survive after confrontation with known observational and theoretical constraints?
3. Can the surviving pieces be written in the language of established braneworld or embedding formalism?
4. If a viable programme remains, what exact calculations would decide it?

The main conclusion is that the original proposal must be substantially pruned to remain viable. Three literal claims do not survive. First, global rigid rotation cannot explain galactic mass discrepancies without violating the observed isotropy of the universe by many orders of magnitude [5, 6]. Second, conventional gravitomagnetic effects are too small to replace dark matter in galaxies [7]. Third, the quantum-nonlocality claim remains a conjectural extension with no mechanism that reproduces standard quantum constraints.

After these eliminations, a residual programme remains. The viable core is *not* that galaxies directly feel a tiny background cosmic vorticity. It is instead that bulk angular momentum may enter the four-dimensional effective theory through the nonlocal Weyl sector, and that this channel can be activated locally by gradients of extrinsic curvature near matter concentrations while remaining suppressed on an exactly Friedmann background. This surviving core is mathematically expressible, but it is not yet a worked halo theory. Moreover, the literature suggests that early-universe geometric dark matter, galactic halo enhancement, and dark-energy-like acceleration belong to different irreducible sectors of the effective theory. The most promising completion is therefore a single higher-dimensional model with multiple sectors rather than a one-term unification.

The paper is organised as follows. Section 2 states the original hypothesis and the pruned version defended here. Section 3 places the proposal in the literature and identifies the nearest prior work. Section 4 develops the relevant effective brane equations and isolates the magnetic-to-electric Weyl channel that survives the initial eliminations. Section 6 analyses the slow-rotation expansion and clarifies what is and is not determined at linear order. Section 7 assesses galactic phenomenology and explains why existing Weyl-fluid fits do not yet constitute a derivation of the proposed spin-coupled halo effect. Section 8 addresses pre-recombination behaviour, including the failure of standard dark radiation to replace cold dark matter and the relevance of linear-dilaton bulk black-hole models. Section 9 asks whether a rotating extension of that scalar-gravity completion can unify the surviving sectors. Section 10 summarises the sharp no-go results and gives decisive observational kill criteria. Section 11 outlines a concrete research programme, and Section 12 briefly comments on the peripheral claims about emergent time and quantum nonlocality.

2. Statement of the hypothesis and its pruned form

2.1. Original form

The original hypothesis may be stated as follows.

> The observed universe is a lower-dimensional boundary or brane associated with a rotating and expanding higher-dimensional geometry. Dark matter is not particulate but an apparent dynamical effect arising from higher-dimensional rotation projected into the observed universe. Dark energy is not a cosmological constant but the effective manifestation of expansion or drift in the extra dimension. Time is emergent from ordered geometric change, and spatial nonlocality in quantum theory reflects hidden higher-dimensional adjacency.

That formulation is conceptually unified, but it conflates several independent tasks: reproducing galactic mass discrepancies; reproducing the pre-recombination matter component required by the cosmic microwave background (CMB); producing late-time acceleration; and generating a theory of time and quantum correlations. The literature does not support treating these tasks as automatically performed by a single kinematical ingredient.

2.2. Three eliminations

The first stage of analysis removes three claims from the core theory.

(i) Global rigid rotation as dark matter: ruled out. If the missing galactic acceleration were literally supplied by a global cosmic rotation, the required present-day angular velocity would be of order $10^{-16}\,\mathrm{s}^{-1}$ for galaxy-scale radii and velocities. This exceeds modern observational bounds on large-scale vorticity by roughly eleven to twelve orders of magnitude. Planck's Bianchi analysis yields $(\omega/H)_0 < 7.6 \times 10^{-10}$ at 95% confidence for the relevant models, while broader tests of isotropy using Planck temperature and polarisation data are also consistent with vanishing anisotropic expansion [5, 6]. The literal rigid-rotation interpretation is therefore not salvageable.

(ii) Pure frame-dragging or gravitomagnetism as dark matter: ruled out. Even if one abandons global cosmic rotation and attempts to generate the missing galactic force from gravitomagnetic corrections, the effect is far too small. Detailed analyses of axisymmetric rotating galaxies in linearised general relativity find gravitomagnetic contributions to circular velocities and vertical support suppressed by roughly six orders of magnitude relative to the Newtonian term, with additional pathologies appearing in attempts to force a fit [7]. This route cannot carry the dark-matter burden.

(iii) Quantum nonlocality as a projection artefact: demoted. The conjecture that Bell-type correlations arise because apparently distant particles remain close in the ambient geometry may be heuristically suggestive, but it is not a mechanism. No version of the proposal considered here reproduces no-signalling, Tsirelson bounds, decoherence, or standard relativistic quantum field theory. The idea may remain a philosophical extension, but it does not belong to the empirical core of the model.

2.3. Pruned formulation

After these eliminations, the proposal that remains worthy of analysis is considerably sharper:

> A higher-dimensional braneworld or embedded cosmology may admit a *locally activated* coupling between bulk angular momentum and brane geometry, mediated by the nonlocal Weyl sector and controlled by extrinsic-curvature gradients. The background Friedmann universe can remain effectively isotropic, while local matter concentrations induce a nontrivial geometric response that may contribute to the effective halo sector. Early-universe geometric dark matter, if present, must arise from a distinct homogeneous scalar sector, and dark-energy-like acceleration must arise from yet another background sector associated with brane motion, extrinsic curvature, or a different equation-of-state branch of the same bulk theory.

This version is no longer a single-mechanism theory. It is a programme for embedding several dark-sector effects in one bulk completion while allowing them to occupy different effective sectors of the four-dimensional theory.

3. Literature position and novelty assessment

3.1. Search strategy and claim of novelty

The literature review underpinning this manuscript was mechanism-specific rather than analogy-based. The relevant search classes were:

1. expanding spherical branes, shell universes, bubble cosmologies, and moving-brane or mirage cosmologies;
2. dark-energy-like effects from embedding, extrinsic curvature, or bulk motion;
3. dark-matter-like effects from Weyl terms, induced gravity, or embedding degrees of freedom;

Table 1: Status of the major claims after applying the strongest currently known objections.

Claim	**Status**	**Reason**
Global rigid rotation explains galactic dark matter	Rejected	Violates CMB isotropy and vorticity bounds by many orders of magnitude [5, 6].
Frame-dragging or gravitomagnetism explains rotation curves	Rejected	Too small by $\mathcal{O}(10^6)$ and dynamically problematic [7].
Quantum nonlocality from higher-dimensional adjacency	Demoted	No mechanism reproducing standard quantum bounds or decoherence.
Bulk spin activates a local geometric response through the Weyl sector	Survives provisionally	Mathematically expressible in brane formalism; not yet a worked halo model [2, 24].
Geometric early-universe matter from an extended bulk sector	Survives in restricted form	Impossible for standard RS dark radiation, but possible in scalar-gravity / linear-dilaton completions [25, 4].
Dark-energy-like acceleration from embedding or bulk-induced background terms	Survives provisionally	Multiple existing mechanisms exist, but not as the same term that gives matter-like behaviour [10, 11, 29].

4. rotating bulks, rotating braneworlds, and slow Kerr–AdS perturbations;
5. scalar-gravity and linear-dilaton five-dimensional models that produce matter-like effective fluids on the brane.

Within the searchable literature considered up to 25 March 2026, no paper was found that already realises the entire proposed combination in a single worked model: *a rotating and expanding closed brane or hypersphere in five dimensions, with galactic dark matter arising specifically from bulk angular momentum projected through local brane geometry, and dark energy arising from extra-dimensional motion or drift.* The literature does, however, contain each of these ingredients separately, and sometimes pairs of them.

That novelty claim must be read cautiously. It is a claim about the indexed and retrievable literature surveyed here, not a proof of nonexistence. What matters for present purposes is that no established calculation already delivers the full mechanism under the required observational constraints.

3.2. Nearest antecedents: expanding spherical branes and shell universes

The closest direct antecedents to the "boundary of an expanding higher-dimensional object" picture are the spherical brane and bubble cosmology papers of Collins and Holdom and the shell-universe papers of Gogberashvili and collaborators. Collins and Holdom studied branes that are either the edge of a single anti-de Sitter bulk or the surface of a vacuum bubble expanding into Schwarzschild or AdS–Schwarzschild space, obtaining Robertson–Walker-like cosmologies from the brane motion itself [8]. Gogberashvili's shell-universe programme similarly treated the universe as a spherical brane and explicitly proposed that the same geometric setting could address both cosmic expansion without a separate dark-energy component and a modified-Newtonian alternative to galactic dark matter [9].

These models are important because they show that the combination "closed brane + extra-dimensional motion + cosmological interpretation" is not new. What is new in the present proposal is the attempt to make bulk angular momentum, rather than shell tension or purely radial motion, the local source of halo-like phenomenology.

3.3. Dark-energy-like behaviour from embedding and extrinsic curvature

A second literature family makes dark-energy-like acceleration a direct effect of embedding geometry. Maia *et al.* interpreted the extrinsic curvature of a Friedmann brane embedded in a five-dimensional constant-curvature bulk as a geometrical dark-energy component and compared the resulting cosmology to XCDM phenomenology [10]. Stern and Xu, working in an extended Regge–Teitelboim embedding framework, showed that a flat Robertson–Walker manifold embedded in five-dimensional de Sitter space can transit from deceleration to acceleration without matter, radiation, or a bare cosmological constant [11]. Related work on variable brane tension has also shown that geometric generalisations of braneworld dynamics can naturally generate evolving effective cosmological terms [12].

These results are the strongest support for the proposition that a dark-energy-like sector can arise geometrically from higher-dimensional embedding. They do *not*, however, imply that the same sector will also mimic pressureless matter or produce a galactic halo law.

3.4. Dark-matter-like behaviour from Weyl terms, induced gravity, and embedding

The most relevant mature line of work on geometric dark matter in braneworlds is the Weyl-fluid literature. Starting from the effective brane Einstein equations of Shiromizu, Maeda, and Sasaki [1], a number of authors explored static spherically symmetric solutions in which the bulk Weyl term behaves as an effective dark source. Mak and Harko showed that the nonlocal bulk contribution could in principle reproduce flat galactic rotation curves without particle dark matter [13]. Harko and Cheng derived the exact exterior metric and the associated dark radiation and dark pressure in the flat-curve regime [14]. Viznyuk and Shtanov analysed the spherically symmetric problem in a model with induced gravity and found a parameter regime where the gravitational field can be described by an effective Weyl fluid playing the role of dark matter [15]. Gergely, Harko, Dwornik, Kupi, and Keresztes later confronted a baryonic+Weyl model with galaxy data and obtained acceptable fits on selected samples [16].

Outside the strictly RS/Weyl-fluid setting, Gurwich and Davidson proposed "artifact dark matter" arising from unified brane gravity [17], while Paston argued from embedding theory that additional geometric degrees of freedom may appear observationally as fictitious matter [3]. Milgrom's brane-MOND picture belongs to the same broad conceptual ecosystem: it is a deliberately heuristic construction in which a nearly spherical brane embedded in a higher-dimensional space gives rise to a MOND-like acceleration scale [18].

This literature establishes something important but limited: higher-dimensional geometry can indeed generate effective source terms that mimic dark matter. What it does *not* yet show is that bulk angular momentum projected through local extrinsic curvature automatically produces the right halo structure.

3.5. Rotating bulks and rotating braneworlds

The explicit literature on rotation is narrower. Guth and Nayeri considered a single brane embedded in a slowly rotating Kerr–AdS bulk and found that, to first order in the bulk angular-momentum parameter, the cosmic fluid on the brane undergoes rigid rotation relative to a Robertson–Walker frame; corrections to the Friedmann equations and the shape of the brane occur only at higher order [19]. Chen, Harko, Kao, and Mak studied rotational perturbations of Friedmann branes and showed that, in expanding models, the rotation generally decays [20, 21]. Steer and Parry, within mirage cosmology, analysed a brane moving in rotating bulk spacetimes and derived new dark-fluid terms in the induced Friedmann equation, including terms controlled by brane angular momentum and bulk structure [22].

These papers are crucial because they show where rotation enters the effective theory and what it tends to do. Their lesson is mixed. Rotation can certainly appear in the effective brane dynamics, but the known explicit solutions mostly produce either rigid background rotation or homogeneous early-time corrections, not a clean late-time halo mechanism localised around galaxies.

3.6. The strongest existing proof-of-principle for geometric dark matter before recombination

The most important recent development for the present proposal is the linear-dilaton or scalar-gravity braneworld programme of Fichet, Megías, and Quirós. In *Cosmological Dark Matter from a Bulk Black Hole*, they showed that the Friedmann equation of a three-brane in a five-dimensional scalar-gravity background automatically contains a term behaving like pressureless matter rather than dark radiation [4]. The matter-like behaviour arises because the bulk black-hole contribution reaches the brane through both the Weyl tensor and the scalar stress tensor, with an exact cancellation of pressures. In *Holographic Fluids from 5D Dilaton Gravity*, they generalised this picture and showed that the effective holographic fluid can interpolate between radiation-like, matter-like, and vacuum-like equations of state as a function of the bulk parameter ν [29]. In *Holographic Dark Matter*, they extended the model into a fuller cosmological scenario in which the bulk black-hole phase is thermodynamically favoured and the late-time contribution redshifts as pressureless matter [30].

This family is the strongest current evidence that a *geometric* dark-matter component can satisfy the early-universe requirement that ordinary RS dark radiation fails to meet. It is therefore the natural place to look for a viable unified completion of the present proposal. The price is that the matter-like term no longer comes from standard RS kinematics alone; it requires an enlarged bulk sector.

3.7. Adjacent same-spirit models and cautionary analogues

Two further neighbouring lines deserve mention. Monjo's hyperconical-universe programme derives both a dark-energy-like term from a projected higher-dimensional cosmology and, more recently,

Table 2: Closest literature families to the components of the proposal. "Support" means support for the *type* of mechanism, not for the full combined model.

Component	Closest literature	Assessment
Expanding closed brane / boundary universe	Collins–Holdom bubble brane; Gogberashvili shell universe [8, 9]	Strong antecedent for the geometric picture.
Dark energy from embedding / extra dimension	Maia *et al.*; Stern–Xu; variable-tension branes [10, 11, 12]	Strong support for a geometric acceleration sector.
Dark matter from geometric effective sources	Mak–Harko; Harko–Cheng; Viznyuk–Shtanov; Gergely *et al.*; Gurwich–Davidson; Paston [13, 14, 15, 16, 17, 3]	Strong support for geometric dark-source phenomenology, but not specifically for a spin-coupled halo law.
Rotation in the bulk / rotating branes	Guth–Nayeri; Chen *et al.*; Steer–Parry [19, 20, 22]	Establishes formal channel for rotation; explicit known solutions do not yet realise the desired phenomenology.
Early-universe geometric matter	Fichet–Megías–Quirós linear-dilaton programme [4, 29, 30]	Strongest proof-of-principle that geometric bulk effects can mimic pressureless matter before recombination.
MOND-like geometry from embedding	Milgrom brane-MOND; Monjo hyperconical models [18, 32]	Conceptually adjacent; not derivations of the present mechanism.

MOND-like galactic phenomenology from a distorted projection of a hyperconical universe embedded in five-dimensional Minkowski space [31, 32]. These papers are among the closest "same-spirit" analogues to the ambition of unifying multiple dark-sector phenomena geometrically, but they are not braneworld slow-rotation models.

At the opposite end, string-motivated dark-bubble cosmology shows that expanding higher-dimensional bubble models can produce induced four-dimensional acceleration and even involve rotating extra-dimensional structures, but recent work has identified serious phenomenological problems, including equivalence-principle violation in the current formulation [33, 34, 35]. That line is therefore useful as a caution: higher-dimensional cosmology can be rich and suggestive without automatically being viable.

4. Effective four-dimensional formalism and the surviving coupling channel

4.1. The standard effective brane equations

The natural starting point is the Shiromizu–Maeda–Sasaki (SMS) projection of the five-dimensional Einstein equations onto the brane [1]. In standard notation, the effective four-dimensional field equations are

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \kappa_4^2 T_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{1}$$

where $\Pi_{\mu\nu}$ is quadratic in the brane energy-momentum tensor, and $\mathcal{E}_{\mu\nu}$ is the projection of the bulk Weyl tensor onto the brane. The latter is the crucial nonlocal term. It carries information about the free gravitational field in the bulk and renders the four-dimensional theory non-closed unless supplementary data or closure conditions are provided [2].

Relative to a timelike four-velocity u^μ, the projected Weyl tensor is conventionally decomposed into an effective energy density, a spatial energy flux, and an anisotropic stress:

$$\mathcal{E}_{\mu\nu} \longleftrightarrow \{U, Q_\mu, P_{\mu\nu}\}, \tag{2}$$

where $Q_\mu u^\mu = 0$, $P_{\mu\nu}u^\nu = 0$, and $P^\mu{}_\mu = 0$. On an exact Friedmann–Robertson–Walker (FRW) background, symmetry forces

$$Q_\mu = 0, \qquad P_{\mu\nu} = 0, \tag{3}$$

so only the scalar part U survives at background level [2]. This simple observation is the formal reason why the surviving programme can, in principle, evade the CMB vorticity objection: the background universe can remain isotropic even if the full theory contains a vector or gravitomagnetic sector.

The nonlocality of $\mathcal{E}_{\mu\nu}$ is reflected in the contracted Bianchi identity, which implies that the divergence of $\mathcal{E}_{\mu\nu}$ is tied to the matter sector via $\Pi_{\mu\nu}$. The four-dimensional observer therefore cannot determine the full Weyl response from brane data alone without extra information about the bulk.

4.2. Why bulk angular momentum must enter through the Weyl sector

If angular momentum is stored in the bulk rather than in an observable global rotation of the brane universe, it cannot appear in Eq. (1) as an ordinary matter term on the brane. It must enter through the nonlocal sector, i.e. through $\mathcal{E}_{\mu\nu}$ or its higher-dimensional evolution. In particular, the magnetic part of the five-dimensional Weyl tensor provides the natural carrier of bulk gravitomagnetic information.

In the covariant $3+1+1$ formalism for brane dynamics, developed by Keresztes and Gergely, the off-brane evolution couples electric and magnetic Weyl components together with extrinsic-curvature variables [23, 24]. Schematically, one has an equation of the form

$$\mathcal{L}_n E_{\mu\nu} = D^\alpha B_{\alpha(\mu\nu)} + 3K^\alpha{}_{(\mu}E_{\nu)\alpha} - KE_{\mu\nu} + \cdots , \tag{4}$$

where n^A is the normal to the brane, D_μ is the brane-covariant derivative, $K_{\mu\nu}$ is the extrinsic curvature, and the omitted terms include additional matter and curvature couplings. The key point is structural: *the electric Weyl tensor felt by brane observers is sourced by derivatives of the magnetic Weyl tensor and by couplings to the extrinsic curvature.*

At the brane itself, the magnetic Weyl term is constrained by the junction conditions and by derivatives of the extrinsic curvature. Again schematically,

$$B_{\mu\nu\alpha}\big|_{\text{brane}} \sim 2D_{[\mu}K_{\nu]\alpha} + \cdots . \tag{5}$$

This is the mathematically relevant form of the surviving intuition: *bulk gravitomagnetic information* *can be transferred into the observable electric Weyl sector through local gradients of the embedding geometry.* In words, bulk angular momentum does not show up on the brane as a direct centrifugal field; it is filtered through the nonlocal Weyl dynamics and activated by inhomogeneous brane geometry.

4.3. The surviving channel stated precisely

The statement that survives the initial eliminations can therefore be made in formal language.

> Let the bulk contain a small but nonzero angular-momentum parameter a. On an exactly homogeneous FRW brane, symmetry suppresses the vector and anisotropic Weyl pieces, leaving no observable background vorticity. Near local matter concentrations on the brane, however, gradients of extrinsic curvature generate a nontrivial magnetic Weyl response, which then sources a correction to the electric Weyl term through Eq. (4). The induced correction contributes to the effective four-dimensional gravitational field and may behave as a local geometric dark source.

This is the channel that deserves detailed investigation. It is already present, at the level of principle, in the established brane formalism. But it is equally important to state what the formalism does *not* yet provide. Equations (1)–(5) do not, by themselves, yield a closed, local, galaxy-scale halo law. The induced term depends on bulk boundary data, closure assumptions, or an explicit solution of the bulk equations. That limitation will be central in the following sections.

5. The sectoral architecture forced by the equations

The literature and formal analysis together suggest that the strongest surviving version of the model is not a single universal correction but a three-sector effective theory:

1. **Sector A: homogeneous scalar matter sector.** A background contribution to the Friedmann equation behaving like pressureless matter, needed before recombination. Standard RS dark radiation fails here, but scalar-gravity or linear-dilaton completions can succeed [4, 29].
2. **Sector B: local spin-activated Weyl sector.** A locally triggered vector or anisotropic response sourced by bulk angular momentum and extrinsic-curvature gradients, potentially relevant for galactic halos and lensing.
3. **Sector C: background acceleration sector.** A late-time dark-energy-like contribution arising from embedding, extrinsic curvature, brane motion, variable tension, or a vacuum-energy-like branch of the same bulk theory [10, 11, 12, 29].

The immediate implication is conceptual but decisive: one may hope for a *single bulk completion* containing all three sectors, but the mathematics does not support the stronger claim that the same effective term does all three jobs.

6. Linearised slow-rotation analysis around an FRW brane

Table 3: Sectoral decomposition of the strongest viable version of the programme.

Sector	**Role**	**Leading character**	**Most relevant existing literature**
A	Pre-recombination matter-like component	Homogeneous scalar sector	Linear-dilaton / scalar-gravity bulk black hole; holographic fluids [4, 29, 30].
B	Local halo / lensing enhancement near galaxies	Vector/off-diagonal at leading order; scalar response only after additional projection or higher order	SMS/Weyl formalism; slow rotating bulks; $3+1+1$ evolution system [1, 24, 19].
C	Late-time acceleration	Background scalar sector	Extrinsic-curvature dark energy; embedding acceleration; vacuum-like holographic branch [10, 11, 29].

6.1. Setup

Consider a slowly rotating five-dimensional bulk with angular-momentum parameter a (or j in the notation of some rotating AdS black-hole solutions), together with a Friedmann brane. Place on the brane a localised, approximately static baryonic mass concentration intended to model a galaxy. The question is whether the spin stored in the bulk can activate a useful effective halo sector near the baryonic source through the chain

$$\text{bulk spin} \;\to\; B_{\mu\nu\alpha} \;\to\; DK \;\to\; E_{\mu\nu} \;\to\; \text{effective four-dimensional gravity.}$$

In existing static spherically symmetric halo analyses, the sector usually studied is the scalar pair (U, P) with $Q_\mu = 0$ [14, 16]. The present mechanism differs precisely in that it attempts to turn on a spin-dependent Q_μ or off-diagonal response through the bulk angular momentum.

6.2. What enters at first order in the bulk spin parameter?

The best explicit guide is the slow Kerr–AdS braneworld calculation of Guth and Nayeri. In their model, the brane metric acquires a $dt\,d\varphi$ cross-term at first order in the bulk angular momentum, and the brane fluid rotates rigidly relative to a Robertson–Walker frame. Corrections to the Friedmann equation and to the brane shape appear only at higher order [19]. This is highly suggestive for the present problem. It indicates that the leading spin-induced effect is not an isotropic scalar density correction but an *off-diagonal, vector-like response.*

The same pattern appears more generally in slow-rotation solutions of dilaton black holes: at linear order in the rotation parameter, the metric is typically modified only through $g_{t\phi}$, while the diagonal background functions remain unchanged up to $\mathcal{O}(a^2)$ [38]. This is not itself a braneworld result, but it strongly supports the following inference.

Leading-order expectation. The spin-activated perturbation in the effective brane theory should enter at $\mathcal{O}(a)$ in the vector or off-diagonal sector (Q_μ-type or $E_{t\phi}$-type), whereas a spherically averaged halo-like scalar correction to U or P is expected only at $\mathcal{O}(a^2)$ unless one allows a genuinely axisymmetric halo response.

This order counting is not a rigorous theorem for the exact configuration of interest, but it follows from symmetry and from the structure of known slow-rotation solutions. A single rotation axis selects a preferred orientation. At linear order, the perturbation is therefore odd under reversal of the spin direction and naturally enters an axial or vector sector. A scalar halo profile, by contrast, is even under $a \to -a$ and is thus generically quadratic.

6.3. Nonlocality and the radial-profile problem

The next question is whether the formalism itself determines the radial dependence of the induced halo term. The answer is no. The brane equations do not close locally. What one obtains is, at best, a nonlocal response of the schematic form

$$\delta E^{\mathrm{spin}}_{\mu\nu}(x) \sim a \int G_{\mu\nu}{}^{\alpha\beta\gamma}(x, x')\, D_\alpha \delta K_{\beta\gamma}(x')\, \mathrm{d}^4x' + \mathcal{O}(a^2), \tag{6}$$

where $G_{\mu\nu}{}^{\alpha\beta\gamma}$ is a bulk-to-brane Green function or kernel determined by the full five-dimensional dynamics and boundary conditions. Equation (6) is schematic, but its message is exact in spirit: the induced response is not a local algebraic function of the baryonic density alone.

This is also the underlying reason why the published Weyl-fluid galaxy models require closure assumptions. In the static spherically symmetric sector, one typically needs an equation of state or an equivalent relation between dark radiation and dark pressure in order to determine the metric and rotation curve [14, 15, 16]. Turning on bulk spin does not remove that freedom; if anything, it enlarges it by opening a vector sector that the static literature usually truncates away.

6.4. Why a MOND-like square-root law is not automatic

A further issue is whether the spin-coupled channel has any intrinsic tendency to generate a MOND-like low-acceleration law,

$$a \sim \sqrt{a_N a_0}, \tag{7}$$

rather than a more conventional power-law correction. There is no known reason to expect such behaviour from the slow-rotation expansion itself.

The reason is structural. A perturbative expansion in the bulk spin parameter a and in weak inhomogeneities naturally generates analytic corrections to the metric or effective sources. MOND's deep-regime law is nonanalytic in the Newtonian field and introduces a distinguished acceleration scale a_0. Nothing in the bare Kerr–AdS or SMS/Weyl formalism yields such a scale automatically. Existing Weyl-fluid galaxy fits obtain approximately flat curves because the effective dark mass grows roughly linearly with radius in the fitted exterior solution, not because a MOND interpolation law emerges from first principles [14, 16]. A MOND-like relation could conceivably emerge from a highly constrained nonlocal closure, but it is not dictated by the formalism used here.

The conclusion of this section is therefore mixed but sharp. The spin-coupled channel is formally real, but its leading-order manifestation is vector-like, its scalar halo response is likely subleading, and its radial profile is not predicted by the four-dimensional effective equations alone.

7. Galactic phenomenology: what the existing literature does and does not provide

7.1. What the Weyl-fluid halo papers actually show

The static braneworld halo papers do demonstrate that extra-dimensional gravity can mimic dark matter in galaxies. In the flat-curve regime, Harko and Cheng derived a metric and associated effective dark radiation and dark pressure for which the induced dark mass can grow roughly linearly with radius, thereby producing nearly constant tangential velocities over an extended region [14]. Gergely *et al.* then showed that a baryonic-plus-Weyl model can fit selected spiral-galaxy rotation curves with acceptable parameter values [16]. Viznyuk and Shtanov found a similar interpretation in a braneworld with induced gravity, emphasising that combined rotation-curve and lensing data could discriminate between effective geometric sources and conventional dark matter [15].

These are nontrivial successes. They show that the effective source language of braneworlds is flexible enough to reproduce some halo-like observables. However, the sector employed in those papers is generally the static, spherically symmetric $Q_\mu = 0$ sector, not the spin-coupled vector channel of interest here. The existing fits therefore cannot be quoted as direct evidence for the present rotating-bulk mechanism.

7.2. The strongest current objection at galactic and cluster scales

The biggest astrophysical challenge for any non-particle explanation of dark matter is not merely rotation curves but the combined requirement of dynamics and lensing, especially in colliding systems. The Bullet Cluster and related merging clusters show that the dominant lensing peaks track the collisionless galaxy component rather than the dissipative X-ray gas [37]. This does not logically exclude all modified or geometric gravity, but it strongly constrains models in which the effective dark source is tied too rigidly to the baryonic gas distribution. A viable geometric alternative must generate localised effective gravitating structures that behave, for lensing purposes, much more like a collisionless halo than like a smoothed response to the dominant baryons.

This matters acutely for the present proposal. If the bulk-spin coupling merely modifies the exterior force law around isolated galaxies without generating a robust lensing sector, it will fail. Any serious version of the programme must therefore compute two potentials: the dynamical potential governing circular motion and the lensing potential governing null geodesics. A geometric halo model that fits one but not the other is not viable.

7.3. What would count as success for the spin-coupled halo sector?

A successful galactic realisation of the surviving channel would need to satisfy all of the following:

1. produce an induced effective source that is negligible on an FRW background but nonzero in the vicinity of matter concentrations;
2. generate a scalar halo response large enough to compete with baryonic gravity at $r \sim 1$–$100\,\mathrm{kpc}$ while remaining compatible with Solar-System and strong-field constraints;
3. yield a radial profile and halo scaling relations that are not purely ad hoc;
4. provide a consistent lensing potential;
5. remain compatible with cluster-merger observations and not simply with isolated-galaxy rotation curves.

At present, none of these has been shown for the specific bulk-spin mechanism. The literature establishes possibility at the level of effective source terms, not an existence proof for the full halo phenomenology.

8. Early-universe constraints and the problem of geometric matter before recombination

8.1. Why standard RS dark radiation is not enough

The requirement imposed by the CMB is not just “extra gravity” at late times. A dark-matter replacement must behave sufficiently like a pressureless clustering component before recombination to reproduce the observed acoustic peak structure, especially the higher peaks. Standard RS braneworld cosmology does not automatically provide such a component. In the simplest case, the bulk black hole induces on the brane a dark-radiation term that scales as a^{-4} and carries an equation of state closer to radiation than dust [2]. Observational constraints from nucleosynthesis and the CMB already restrict the allowed contribution of such dark radiation to be subdominant [26]. Koyama showed that dark-radiation perturbations induce isocurvature effects, with the resulting CMB spectrum additionally shaped by Weyl anisotropic stress; this is not equivalent to replacing cold dark matter with a new pressureless component [25].

Therefore, any version of the present proposal that relies only on standard RS/Weyl dark radiation for the early universe is effectively ruled out as a complete dark-matter substitute.

8.2. The generalized Weyl-fluid loophole

A more flexible possibility arises when one allows brane-bulk energy exchange or more general bulk matter. In Pal’s generalised brane cosmology, the Weyl-fluid density scales as

$$\rho_* \propto a^{-(4-\alpha)}, \tag{8}$$

so that the effective component need not redshift exactly like radiation [27]. In such models one can obtain slower dilution and, at least in perturbation theory, growing Weyl-fluid modes that may assist structure formation. Gergely, Keresztes, and Szabó similarly explored models with $\alpha = 0, 2, 3$, where the Weyl-fluid component redshifts more slowly than ordinary dark radiation and can remain relevant at late times [28].

This loophole is important because it demonstrates that geometry-induced effective fluids need not be radiation-like in every higher-dimensional completion. But it is not yet a full solution to the CMB problem. The generalised Weyl-fluid papers do not provide a demonstration that the observed CMB acoustic structure can be reproduced with a dominant geometric component replacing standard cold dark matter throughout the pre-recombination epoch.

8.3. Linear-dilaton bulk black holes as a stronger solution

The linear-dilaton braneworld models of Fichet, Megías, and Quirós go further. In that framework, the bulk black-hole contribution to the brane Friedmann equation behaves like pressureless matter rather than dark radiation because the Weyl and scalar contributions conspire to cancel the effective pressure [4]. This is a crucial proof-of-principle result: it shows that a geometric or holographic matter sector can satisfy the *equation-of-state* requirement for cold dark matter, at least at background level.

The subsequent generalisation to a broader class of five-dimensional dilaton-gravity backgrounds sharpened the point. The effective brane fluid can interpolate continuously between radiation-like, matter-like, and vacuum-like behaviour depending on a bulk parameter ν [29]. In other words, a single higher-dimensional theory may contain the raw material for both matter-like and dark-energy-like sectors, but the sectors remain distinct in their effective behaviour.

Finally, the 2026 extension to “holographic dark matter” added a fuller cosmological scenario in which a thermodynamically favoured bulk black-hole phase is populated by freeze-in energy transfer from the brane and contributes a late-time pressureless component consistent with current bounds on the five-dimensional Planck scale [30]. This makes the linear-dilaton model the strongest currently available candidate for Sector A in Table 2.

8.4. Interim verdict on the early universe

The early-universe question therefore has a clear answer.

- **Standard RS/Weyl dark radiation: insufficient.**
- **Generalised Weyl-fluid models with energy exchange: suggestive but incomplete.**
- **Extended scalar-gravity or linear-dilaton bulk completions: viable in principle and currently the best route.**

This already forces an important split in the original proposal: any realistic unified model must go beyond the simplest rotating RS picture if it aims to satisfy the pre-recombination matter requirement.

9. Can a rotating linear-dilaton completion unify the surviving sectors?

9.1. The key question

The natural next step is to ask whether the strongest surviving early-universe mechanism—the linear-dilaton bulk black-hole model—can be supplied with a rotating bulk generalisation that also produces a local spin-activated halo enhancement while retaining a dark-energy-like background sector. In concrete terms, one asks whether a rotating black hole in an extended five-dimensional scalar-gravity background can simultaneously generate:

1. a homogeneous pressureless matter-like term before recombination;
2. an additional galactic-scale correction activated by bulk angular momentum through DK and the Weyl sector;
3. a background acceleration sector from embedding, brane motion, or a vacuum-like branch of the same theory.

9.2. Why the matter-like pressure cancellation likely survives at first order in spin

The decisive structural observation is that, in known slowly rotating dilaton black-hole solutions, the leading perturbation is off-diagonal: the first-order correction is carried by $g_{t\phi}$, while the scalar background functions typically remain unchanged up to $\mathcal{O}(a^2)$ [38]. If a rotating extension of the Fichet–Megías–Quirós background shares this property, then the pressure cancellation responsible for the effective matter term should survive in the isotropic scalar sector at $\mathcal{O}(a)$. Rotation would then add a new vector or anisotropic correction without immediately spoiling the existing matter-like background.

This is not a proved result for the exact linear-dilaton braneworld geometry. It is a strongly motivated inference based on the general structure of slow-rotation solutions. But it has a major consequence: a rotating extension of the linear-dilaton model is not obviously self-contradictory.

9.3. Where the split is forced

Even if the matter-like cancellation survives, the hoped-for one-term unification still does not emerge. The reason is representation-theoretic as much as dynamical. The pressureless matter term in the Fichet–Megías–Quirós model is a *homogeneous scalar* contribution to the Friedmann equation. The leading spin response is a *vector or off-diagonal* sector. A background acceleration term is again a *scalar* sector but with vacuum-like rather than dust-like behaviour. These sectors transform differently and therefore do not combine into a single universal effective source.

This is the precise point at which the mathematics forces the conceptual split. One may still pursue a *single bulk theory* containing all of them, but one should no longer expect a single correction term to explain dark matter at all scales, dark energy, and local halos simultaneously.

9.4. A danger from known rotating-bulk cosmologies

A second issue is temporal behaviour. In Steer and Parry's mirage-cosmology analysis of rotating bulk spacetimes, the induced Friedmann equation contains bulk-dependent dark-fluid terms such as a^{-6} and a^{-10} in addition to the more familiar terms [22]. Those contributions are largest at small scale factor and thus risk modifying the early universe precisely where the matter-like behaviour of the linear-dilaton mechanism is most valuable. This shows that naively adding rotation to a bulk cosmology can easily contaminate the early-universe sector.

Therefore, for the unified programme to work, the rotating sector must be arranged so that its homogeneous contribution is either absent or strongly suppressed on the FRW background, becoming visible only when local inhomogeneity activates it. That is precisely the role assigned to the $B \to E$ through DK channel. But achieving this in an explicit solution remains an open problem.

9.5. Tractability of the calculation

The good news is that the first stage of the unified calculation is tractable. One need not solve the full nonlinear rotating scalar-gravity system to make progress. A standard and realistic programme is:

1. start from the static linear-dilaton bulk black-hole background of Fichet, Megías, and Quirós;
2. introduce a stationary axisymmetric perturbation,

$$g_{t\phi} = a\,\omega(r,\theta) + \mathcal{O}(a^2), \tag{9}$$

   together with any required perturbation of the dilaton or gauge fields;
3. solve the five-dimensional Einstein–dilaton equations to first order in a;
4. project the solution onto the brane and verify whether the scalar matter-like term is unchanged at $\mathcal{O}(a)$ while new Q_μ and $P_{\mu\nu}$ pieces appear;
5. only then add a localised baryonic perturbation on the brane and compute the nonlocal response.

This first paper is entirely feasible as a slow-rotation perturbation problem. The more difficult second paper is the halo calculation itself, because the nonlocal Weyl response requires either a bulk Green-function construction or a numerical solution of the axisymmetric bulk perturbation with a localised brane source. That difficulty is serious but not prohibitive. It is the natural make-or-break calculation for the programme.

10. Fatal objections, hard constraints, and decisive observational tests

A useful research programme must know in advance what would kill it. The present one already inherits several hard constraints.

10.1. Large-scale isotropy and vorticity

Any version that revives literal global rotation as the source of galactic anomalies is dead on arrival. The background universe is observed to be extremely isotropic. The Planck Collaboration derived, for Bianchi analyses of the relevant type, a bound of $(\omega/H)_0 < 7.6 \times 10^{-10}$ [5], while the more general analysis of Saadeh *et al.* found anisotropic expansion disfavoured at odds of roughly $1.2 \times 10^5 : 1$ [6]. This is why the surviving programme is forced into a background-suppressed, locally activated form.

10.2. Conventional gravitomagnetism

Any version in which the observable halo force comes directly from ordinary four-dimensional frame dragging is also excluded. Lasenby, Hobson, and Barker showed that such effects are $\mathcal{O}(10^{-6})$ too small to explain observed galaxy rotation curves and do not provide the required vertical support [7]. The surviving model must therefore rely on an effective induced source, not on raw linearised GR gravitomagnetism.

10.3. Pre-recombination matter and the acoustic peaks

Any completion that reduces in the early universe to standard RS dark radiation rather than pressureless matter is not a full dark-matter alternative. The Planck 2018 results remain very well fit by the standard ΛCDM parameter set, including $\Omega_c h^2 \simeq 0.120$ [36]. A viable geometric substitute must reproduce the role played by this matter component in the acoustic peak structure and structure growth. Standard RS dark radiation fails this requirement; extended bulk completions may pass it but have not yet been demonstrated to do so in the specific rotating scenario of interest.

10.4. Lensing and cluster mergers

A model that matches isolated-galaxy rotation curves but not lensing or colliding clusters is not viable. The Bullet Cluster remains the canonical benchmark [37]. The geometric programme would be falsified if the induced halo sector cannot produce spatially localised effective gravitating structures with the required lensing behaviour.

10.5. Phenomenological consistency of the bulk completion

Finally, one must not assume that every elegant higher-dimensional construction is phenomenologically safe. The recent critique of dark-bubble cosmology by Basile, Borys, and Masias is a reminder that bulk completions can fail for reasons far removed from their original cosmological motivation, such as equivalence-principle violation [35]. Any rotating linear-dilaton extension pursued here must therefore be checked not only against cosmology but also against local tests of gravity and Standard-Model couplings.

Table 4: Kill criteria for the surviving research programme.

Test	Potential outcome	Consequence
Slow-rotation extension of the linear-dilaton model	Matter-like scalar sector spoiled already at $\mathcal{O}(a)$	Programme fails as a unified early+late scenario.
Localised baryonic perturbation in rotating bulk	Induced scalar halo correction remains negligible or purely axisymmetric with the wrong magnitude	Spin-coupled halo mechanism fails.
Lensing calculation	Dynamical and lensing potentials disagree with observed galaxy or cluster data	Geometric halo sector is not viable.
Cosmological perturbation analysis	Effective matter sector fails CMB acoustic-peak or structure-growth tests	Extended bulk completion fails as a replacement for cold dark matter.
Background anisotropy analysis	Rotating sector leaks into FRW background beyond isotropy bounds	Local-activation ansatz fails.
Equivalence principle / local gravity tests	Rotating bulk completion induces unacceptable Standard-Model or weak-equivalence-principle violations	Theory ruled out irrespective of cosmological fits.

11. Concrete research roadmap

The strongest value of the present analysis is not that it proves the model, but that it isolates a tractable sequence of calculations. The following roadmap is both positive and falsifiable.

11.1. Paper I: rotating extension of the linear-dilaton bulk black hole

Objective. Determine whether the pressureless-matter sector of the Fichet–Megías–Quirós model survives slow rotation.

Method. Construct a stationary axisymmetric perturbation of the five-dimensional Einstein–dilaton background to first order in the rotation parameter. Compute the induced brane Friedmann equation and identify the decomposition of the effective stress tensor into scalar, vector, and anisotropic sectors.

Success condition. The matter-like scalar contribution remains intact at $\mathcal{O}(a)$, while the new spin contribution appears only in vector or anisotropic sectors.

Failure mode. The dust-like sector is immediately spoiled, or rotation generates a forbidden homogeneous anisotropy.

11.2. Paper II: localised brane source and halo activation

Objective. Compute the response of the Weyl sector to a localised baryonic perturbation on the brane in the rotating bulk.

Method. Solve the relevant bulk perturbation equations either by a Green-function construction or by a numerical axisymmetric boundary-value problem. Derive the induced correction to $E_{\mu\nu}$ on the brane and extract the effective dynamical and lensing potentials.

Success condition. The induced scalar halo sector becomes significant near galaxies while remaining negligible on the FRW background. The correction has the right order of magnitude and does not require implausible fine-tuning.

Failure mode. The leading response remains purely vectorial, or the scalar halo is too small or of the wrong sign.

11.3. Paper III: cosmological perturbations and early-universe tests

Objective. Determine whether the extended bulk completion reproduces the role of cold dark matter in the CMB and structure growth.

Method. Develop the scalar and vector perturbation theory around the matter-like background sector, keeping the nonlocal Weyl contributions and any dilaton degrees of freedom. Compare with CMB and matter-power-spectrum observables.

Success condition. The matter-like sector behaves sufficiently like pressureless matter up to recombination and through linear structure formation.

Failure mode. The effective fluid carries too much pressure, anisotropic stress, or isocurvature power.

11.4. Paper IV: lensing and cluster-merger phenomenology

Objective. Test whether the local geometric halo sector behaves like the observed dark component in lensing and cluster mergers.

Method. Compute null geodesics in the induced metric, infer weak-lensing maps, and compare with galaxy-scale and cluster-scale data.

Success condition. The theory reproduces both rotation curves and lensing without reintroducing hidden particle halos by hand.

Failure mode. The model fits only dynamics but not lensing, or produces diffuse geometric responses where the data require collisionless halo-like structure.

11.5. A concise statement of the programme

The viable research programme can now be stated succinctly:

> *Seek a rotating five-dimensional scalar-gravity or linear-dilaton bulk completion in which a homogeneous matter-like brane sector arises from the bulk black hole, while a subleading but locally activated spin-dependent Weyl response is sourced by extrinsic-curvature gradients near matter concentrations, and a separate background sector accounts for late-time acceleration.*

This is significantly narrower than the original one-premise unification, but it is a genuine, mathematically structured target rather than a purely qualitative speculation.

12. Peripheral claims: emergent time and quantum nonlocality

Two ideas from the original proposal remain worth recording, though not as part of the core empirical theory.

12.1. Emergent time

The suggestion that time is not fundamental but abstracted from ordered change has serious antecedents in relational mechanics and shape dynamics [39, 40]. If the braneworld proposal is ever developed into a Hamiltonian or constrained-dynamical framework, the emergent-time claim could be recast more sharply using relational clocks, York time, or shape-dynamical variables. At present, however, it does no explanatory work for the dark sector. It should therefore be treated as a conceptual interpretation of the geometry rather than as a tested component of the cosmological model.

12.2. Quantum nonlocality

The higher-dimensional-adjacency intuition may inspire future work, but it is not yet physics in the relevant sense. No derivation has been given of Bell correlations, no-signalling, Tsirelson bounds, measurement statistics, or the classical limit. For that reason the nonlocality claim should be omitted from the core paper presenting the cosmological mechanism. At most, it may appear in the final discussion as a long-term speculative extension contingent on the success of the geometric programme at the gravitational level.

13. Conclusion

The central question posed in this manuscript was whether there exists a viable and rigorous version of a rotating embedded braneworld cosmology in which geometric structure accounts for both dark matter and dark energy. The answer is neither a simple yes nor a simple no.

The original strongest form of the proposal fails. Global rigid rotation cannot explain galactic dark matter without violating CMB isotropy bounds by orders of magnitude. Conventional four-dimensional gravitomagnetism is far too weak to replace dark matter in galaxies. The quantum-nonlocality extension is presently non-mechanistic. If the proposal were tied irrevocably to those claims, it would be dead.

Yet the entire programme is not dead. Once those claims are removed, a mathematically meaningful channel remains. In braneworld language, bulk angular momentum belongs to the magnetic or nonlocal Weyl sector, and the established $3+1+1$ formalism shows that this sector can feed the observable electric Weyl term through gradients of the extrinsic curvature. Because FRW symmetry eliminates the vector and anisotropic pieces of the Weyl tensor at background level, the theory can in principle remain cosmologically isotropic while still admitting locally activated corrections around inhomogeneities. That is the key surviving insight.

However, the literature forces a further conceptual refinement. There is no evidence for a single effective term that simultaneously explains pre-recombination dark matter, galactic halos, and late-time acceleration. The strongest viable formulation is instead a *single bulk completion with multiple effective sectors*: a homogeneous scalar matter sector for the early universe; a distinct local spin-activated Weyl sector for possible galactic enhancement; and a separate scalar background sector for acceleration. In the present state of the literature, the linear-dilaton bulk black-hole models of Fichet, Megías, and Quirós provide the strongest candidate for the first of these sectors, while the rotating-braneworld and $3+1+1$ formalisms identify the right channel for the second. The third already has multiple geometric antecedents in embedding and extrinsic-curvature cosmology.

The unresolved bottleneck is now precise. The theory lives or dies on whether the spin-coupled $B \to E$ through DK channel generates a scalar halo response of the right magnitude, profile, and lensing behaviour when a localised baryonic source is placed on the brane. That calculation has not been done in the literature surveyed here. It is difficult, but it is tractable enough to define a real research programme.

The most honest final verdict is therefore this: *there is no completed unified model yet, but there is a narrowed, mathematically well-posed, and genuinely falsifiable path forward.* That path no longer defends the original one-term unification. It instead proposes that one five-dimensional rotating scalar-gravity completion may host several distinct effective sectors, each responsible for a different part of the dark sector problem. Whether nature actually uses such a construction remains entirely open. But unlike the discarded versions of the hypothesis, this refined programme is not obviously inconsistent with either the formalism or the existing observational constraints.

A. Order-of-magnitude no-go for literal rigid rotation

It is useful to spell out the order-of-magnitude reason why the literal global-rotation explanation fails. Suppose one tries to generate the observed missing acceleration in a galaxy through a centrifugal term $a_{\mathrm{rot}} \sim \Omega^2 r$. Taking a characteristic outer-galaxy radius $r \sim 10\,\mathrm{kpc} \approx 3 \times 10^{20}\,\mathrm{m}$ and a required excess acceleration $a_{\mathrm{rot}} \sim 10^{-10}\,\mathrm{m\,s^{-2}}$, one obtains

$$\Omega \sim \sqrt{\frac{a_{\mathrm{rot}}}{r}} \sim 6 \times 10^{-16}\,\mathrm{s^{-1}}. \tag{10}$$

A Coriolis-type estimate, $a \sim 2\Omega v$ with $v \sim 200\,\mathrm{km\,s^{-1}}$, gives a similar scale,

$$\Omega \sim \frac{a}{2v} \sim 2.5 \times 10^{-16}\,\mathrm{s^{-1}}. \tag{11}$$

Now compare with the Planck Bianchi bound $(\omega/H)_0 < 7.6\times10^{-10}$ [5]. Using $H_0 \approx 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, i.e. $H_0 \approx 2.18 \times 10^{-18}\,\mathrm{s^{-1}}$ [36], one finds

$$\omega \lesssim 1.7 \times 10^{-27}\,\mathrm{s^{-1}}, \tag{12}$$

which is roughly eleven to twelve orders of magnitude smaller than what the galactic explanation would require. This mismatch is so large that it cannot be repaired by modest model variations.

B. Why the leading spin response is expected to be vector-like

Let a denote the bulk spin parameter. Under reversal of the spin direction, $a \to -a$. A scalar halo density correction must be even under this transformation, whereas an axial or off-diagonal effect may be odd. In a slow-rotation expansion,

$$g_{AB} = g^{(0)}_{AB} + a\,h^{(1)}_{AB} + a^2 h^{(2)}_{AB} + \cdots\,, \tag{13}$$

symmetry therefore allows $h^{(1)}_{AB}$ to appear in axial components such as $g_{t\phi}$ but not in a spin-averaged scalar halo profile. This expectation is realised in known slowly rotating black-hole solutions in which the leading correction is off-diagonal [19, 38]. The scalar response relevant for a quasi-spherical halo is therefore generically deferred to $\mathcal{O}(a^2)$ unless axisymmetry is retained explicitly in the halo phenomenology.

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